

Fluctuations of the gravitational constant induced by primordial bubbles

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Abstract

We consider the classical fluctuations of the gravitational constant generated by bubbles in the inflationary universe. For extended inflation, we demonstrate numerically how and how large fluctuations are produced during bubble expansion. The amplitude of the fluctuations depends on the Brans-Dicke parameter ω : if ω is of the order of unity, the amplitude becomes of the order of unity within one Hubble expansion time; if ω is large (say, $\omega = 1000$), the growth rate of the fluctuations is small, but it keeps growing without freezing during inflation. We also discuss some astrophysical implications of our results.

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Particle physics predicts that the universe experienced many phase transitions in its early history. If any of these phase transitions is first order, the universe changes its phase from false vacuum to true vacuum through the creation, expansion and collision of bubbles. Old inflation [1] is based on a super-cooled first-order phase transition. In the inflationary scenario, the universe expands exponentially with time before the transition, thereby solving the horizon, flatness, and monopole problems. However, it turns out that this exponential expansion is too rapid to permit a transition from false vacuum to true vacuum via percolation of true vacuum bubbles [2].

Extended inflation [3] revived the idea of old inflation by using the Brans-Dicke theory instead of the Einstein theory. The Brans-Dicke field decelerates the expansion of the universe so that true vacuum bubbles can coalesce, thus ending the phase transition that drives inflation. This model provides an interesting hypothesis that the large-scale structure of galaxy distribution is generated by primordial bubbles [4]. Many discussions about extended inflation have been made in past years. La *et al.* and Weinberg [5] estimated volume fraction of bubbles and found a difficulty: the constraint from the isotropy of the cosmic microwave background requires the Brans-Dicke parameter $\omega < 25$, which contradicts the lower limit $\omega > 500$ from the time-delay experiments [6]. This problem is called a "big-bubble problem" because many big bubbles cause large-scale inhomogeneity. Thus interest shifted to other extended models based on general scalar-tensor theories [7]-[9].

In discussing density fluctuations in extended inflation or other extended models, several authors have estimated volume fraction of bubbles as we mentioned above, or quantum fluctuations of the gravitational constant (the Brans-Dicke field) [10]. In this paper, we investigate the classical fluctuations of the gravitational constant generated during bubble expansion. It was shown that, under the thin-wall approximation, the Brans-Dicke field must be inhomogeneous inside a bubble from the consistency of the junction conditions [11]. However, it is not clear how and how large fluctuations are really generated. If these fluctuations are not small, we should consider it for the constraint for the models, such as the structure formation process and the measurements of the gravitational constant.

There is an alternative scenario of inflation which is accompanied by bubble nucleation: one-bubble inflation [12]. This model is distinguishable from extended inflation or its generalized version: the second slow-rollover inflation occurs inside a nucleated bubble, and our observable universe is entirely contained in one bubble. If we assume the $O(4)$ -symmetric bubble, the interior of the bubble can be a homogeneous and isotropic open universe [13]; this model may account for the universe model with $\Omega_0 \lesssim 0.1$, which is supported by increasing observations. Although the fluctuations of the gravitational constant may not be relevant to this model, it is interesting to study the fluctuations inside a bubble in a similar way.

To begin with, we explain briefly why fluctuations of the gravitational constant are generated in the inflationary universe even at a classical level. Let us consider the system which is described by the action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} \left\{ \Phi \mathcal{R} - \frac{\omega(\Phi)}{\Phi} (\nabla \Phi)^2 \right\} - U(\Phi) - \frac{1}{2} (\nabla \psi)^2 - V(\psi) \right], \quad (1)$$

where Φ is a Brans-Dicke-like scalar field which is normalized to be unity at the present epoch, and ψ is a Higgs-like inflaton field. The field equation for Φ is derived from (1):

$$\square \Phi = \frac{1}{2\omega(\Phi) + 3} \left[-8\pi G \left\{ (\nabla \psi)^2 + V(\psi) \right\} - \omega'(\Phi) (\nabla \Phi)^2 + U'(\Phi) \right]. \quad (2)$$

Just looking at (2), we can understand that the inhomogeneity of ψ causes that of Φ : even if Φ and ψ are homogeneous at the beginning of inflation, bubble nucleation, which is realized by the inhomogeneization of ψ , makes Φ inhomogeneous inside a bubble. Because most bubbles become superhorizon scale at the end of inflation, the fluctuations of the gravitational constant, which trace these bubbles, cannot disappear soon after inflation.

We may note, in passing, that the inhomogeneization inside a bubble occurs in generic systems. After a conformal transformation and the introduction of a new scalar field ϕ , many scalar-tensor theories have the Lagrangian form [14]

$$S = \int d^4x \sqrt{-g} \left[\frac{\mathcal{R}}{2\kappa^2} - \frac{1}{2} (\nabla \phi)^2 - W(\phi) - \frac{1}{2} e^{-\gamma\kappa\phi} (\nabla \psi)^2 - e^{-\beta\kappa\phi} V(\psi) \right], \quad (3)$$

where β and γ are dimensionless coupling constants, and $\kappa^2 \equiv 8\pi G$. The field equation for ϕ is written as

$$\square \phi = -\frac{\gamma\kappa}{2} e^{-\gamma\kappa\phi} (\nabla \psi)^2 - \beta\kappa e^{-\beta\kappa\phi} V(\psi) + W'(\phi). \quad (4)$$

This equation also indicates that the inhomogeneity of ψ causes that of ϕ . Thus we see that the inhomogeneity inside a bubble is a generic property in generalized Einstein theories or in multiple scalar field theories.

In what follows we numerically investigate how and how large fluctuations of Φ are really generated in the inflationary era. In order to solve the field equations, we must specify a gravitational theory, i.e., the functions of $\omega(\Phi)$ and $U(\Phi)$. Here we simply assume $\omega(\Phi) = \text{constant}$ and $U(\Phi) = 0$, which corresponds to extended inflation. Although our assumption is just for simplicity, our results will be also applied to some of the models with variable ω , so-called hyperextended inflation [8,9]. Steinhardt and Accetta [8] and Garcia-Bellido and Quiros [9] independently proposed similar scenarios: $\omega(\Phi)$ is small and almost constant during inflation and diverges as Φ approaches $\Phi_0 (= 1)$. As long as we discuss their models, the assumption of constant ω during inflation is justified.

We assume a spherically symmetric spacetime:

$$ds^2 = -dt^2 + A^2(t, r)dr^2 + B^2(t, r)r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (5)$$

As a potential of the inflaton field, we adopt the quartic potential which was used in [15]:

$$V(\psi) = (\psi - m)^2 \{3(\epsilon + 1)\psi^2 + 2\epsilon m\psi + \epsilon m^2\}. \quad (6)$$

Free parameters are ϵ and m : ϵ determines the shape of the potential and the ratio of m to the Planck mass m_{Pl} implies the strength of gravity. We depict some shapes of the potential in Fig. 1.

As an initial configuration of the inflaton field, $\psi(t = 0, r)$, we give a static bubble solution in a flat spacetime by numerically solving the field equation. As for the Brans-Dicke field, we assume $\Phi(t = 0, r)$ to be homogeneous. We also suppose that $\partial\psi/\partial t(t = 0, r) = \partial\Phi/\partial t(t = 0, r) = 0$. To solve the time-dependent field equations, we use a finite difference method with 1000 meshes; the details of the method were shown in the Appendix of [16]. The Hamiltonian constraint equation is used for checking the numerical accuracy; through all the calculations executed here, the errors are always less than a few percent.

Our numerical results are summarized in Figs. 2–5. We normalize the time scale and the spatial scale by $\chi^{-1} \equiv \{8\pi GV(0)/3\Phi(0)\}^{-\frac{1}{2}}$, which corresponds to the horizon scale at the nucleation time. In Fig. 2 we show examples of the time-evolution of ψ and Φ . We see that the fluctuations of Φ are really generated as a bubble expands.

Here we define a quantity which corresponds to the amplitude of the fluctuations as

$$\Delta(t) \equiv \frac{\Phi(t, r_{\text{max}}) - \Phi(t, 0)}{\Phi(t, r_{\text{max}})}, \quad (7)$$

where r_{max} is the outer numerical boundary. We utilize $\Delta(t)$ to present the following results. In Fig. 3(a) we draw $\Delta(t)$ for various ω . As we can expect from (2), $\Delta(t)$ at fixed t is almost proportional to $1/\omega$: if $\omega = 1$, the amplitude becomes of the order of unity within one Hubble expansion time; if $\omega = 1000$, the growth rate of the fluctuations is small, but $\Delta(t)$ goes on increasing without freezing until one bubble collides with another.

Samuel and Hiscock [15] investigated Euclidean solutions of an $O(4)$ -symmetric bubble and found their configurations for m/m_{Pl} of the order of unity are quite different from those of ordinary bubble solutions. Here we pay attention to the behavior of $\Phi(t, r)$ for large m/m_{Pl} . Figure 4 shows the evolution of $\Phi(t, r)$ for $m/m_{\text{Pl}} = 0.5$. The behavior of $\Phi(t, r)$ is quite different from that for small m [cf. Fig. 2 (b)]. In Fig. 4(b) we draw $\Delta(t)$ for $m/m_{\text{Pl}} = 0.1, 0.2$, and 0.5 . As m/m_{Pl} becomes larger, the fluctuations of Φ become larger. We can interpret our results as follows. In the case of large m/m_{Pl} , the viscosity term of

$\partial\Phi/\partial t$ in the right-hand side of (2) is dominant to spatial derivatives, and therefore the homogeneization process is slowed.

We also investigate the dependence of the fluctuations of Φ on the potential shape. We draw $\Delta(t)$ for several ϵ in Fig. 5. The potential for $\epsilon = 0.05$ corresponds to the case of a thin-wall bubble, and $\epsilon = 0.5$ to a thick-wall bubble. The temporal behavior of $\Delta(t)$ for small ϵ (thin-wall bubble) is violent, but the eventual behavior depends little on ϵ .

So far we have numerically analyzed the evolution of a bubble in extended inflation and see how and how large fluctuations of the Brans-Dicke field are generated. In order to understand the astrophysical implications of our results, we also have to analyze their subsequent evolution after the phase transition. Because most fluctuations are nonlinear and gravity is not described by the Einstein theory, however, such analysis is not easy: another simulation or some sophisticated approaches are needed. In the rest of the paper, we offer discussion on the rough implications to some inflationary models.

In the scenario of extended inflation or hyperextended inflation, all bubbles are nucleated with the subhorizon size and cross outside the horizon during inflation. After the phase transition, matter trace the bubbles—we call such bubblelike distribution after inflation “voids”—and reenter again during the radiation-dominated era or the matter-dominated era. Bubbles nucleated the earliest cross outside the horizon first, reenter last, and become the largest voids at present. Although we do not know the exact evolution of voids, we may suppose that homogeneization inside the void does not occur effectively until the void reenters the horizon. Because astrophysical-sized voids (30–100Mpc) reenters the horizon at $z \approx 10^4$ to 10^3 , the inhomogeneity of the gravitational constant affects the evolution of voids during the radiation-dominated universe. In relation to the big-bubble problem of extended inflation, Vadas [17] studied thermalization of superhorizon voids in the radiation-dominated universe and found homogeneization can occur in a much shorter time than previously thought. This result indicates that there remains a possibility that the big-bubble problem is resolved; it is worth studying further. In order to analyze the thermalization process more precisely, it is important to take the inhomogeneity of the gravitational constant into account.

Let us discuss the following question: is it possible to observe the fluctuations of the gravitational constant at present? According to the linear perturbation analysis in the matter-dominated Brans-Dicke universe [18], the decay rate of the amplitude of Φ is approximately proportional to (scale factor) $^{-\frac{3}{4}}$ for $\omega \gg 1$. This indicates that, if $\delta\phi/\phi < 1$ when a void enters the horizon and the linear perturbation theory can be applied, the present fluctuations of Φ inside astrophysical-sized voids are too small to be observable. However, if the fluctuations are nonlinear at the time, it may be possible that a large inhomogeneity still

remains although the amplitude cannot be estimated without further analysis. Small-scale fluctuations caused by small bubbles are also interesting to study. While Turner and Wilczek [19] pointed out the detectability of the gravitational waves produced by bubble collisions in extended inflation, small-scale bubbles could be a source of “scalar-type” gravitational waves which can be detected by a laser interferometer.

Finally, we make some comments on one-bubble inflation. If we consider a single scalar field in the Einstein theory, which is described by the $O(4)$ -symmetric bounce solution [13], the interior of the bubble can be a homogeneous, isotropic, and open universe. We may understand that the high symmetry allows the inside spacetime to have a homogeneous and open slicing by a miracle. On the other hand, if we suppose two scalar fields or a modified Einstein gravity, $O(4)$ symmetry is lost and the inside must be inhomogeneous. Our analysis gives a generic picture that in such a theory the fluctuations grow during bubble expansion. Unless the amplitude of the fluctuations are negligibly small at the end of inflation, the inhomogeneity can be observed at present because it is still on a super-horizon scale. Recently, Linde and Mezhlumian [20] proposed some models with two scalar fields, taking into account the condition that the inhomogeneity inside a bubble should be negligible. It may also be important to consider the classical fluctuations in models with a modified Einstein gravity such as proposed in [21].

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Figure Captions

FIG. 1. The potential of the inflaton field we used in our analysis. We depict the shapes for $\epsilon = 0.05, 0.1$, and 0.5 .

FIG. 2. Examples of the evolution of (a) ψ and (b) Φ . We see how the fluctuations of Φ are generated as a bubble expands.

FIG. 3. Dependence of the fluctuations of Φ on ω . We draw the amplitude $\Delta(t)$ for $\omega = 1, 10, 100$, and 1000 . We set $\epsilon = 0.1$ and $m = 0.1$. $\Delta(t)$ at fixed t is almost proportional to $1/\omega$, but it goes on increasing even for large ω .

FIG. 4. Dependence of the fluctuations of Φ on m/m_{Pl} . We set $\omega = 10$ and $\epsilon = 0.1$. In (a) we show the evolution of $\Phi(t, r)$ for $m/m_{Pl} = 0.5$. The behavior of $\Phi(t, r)$ is quite different from that for small m . In (b) we draw $\Delta(t)$ for $m/m_{Pl} = 0.1, 0.2$, and 0.5 . As m/m_{Pl} is larger, the fluctuations of Φ become larger.

FIG. 5. Dependence of the fluctuations of Φ on the potential shape. We draw $\Delta(t)$ for $\epsilon = 0.05, 0.1$, and 0.5 . We set $\omega = 10$ and $m = 0.1$. Although the temporal behaviors of $\Phi(t, r)$ are quite different, the eventual behavior does not depend on the potential shape.

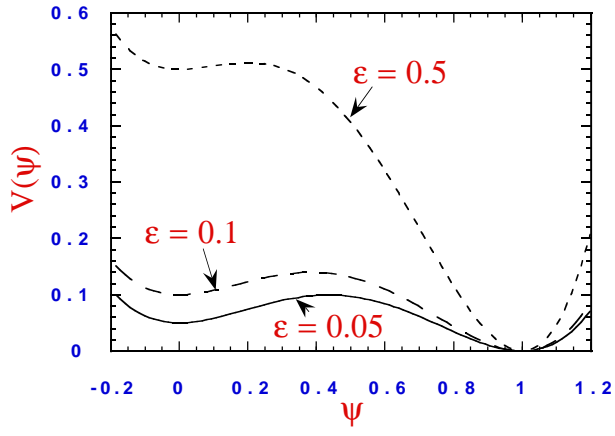


Fig.1

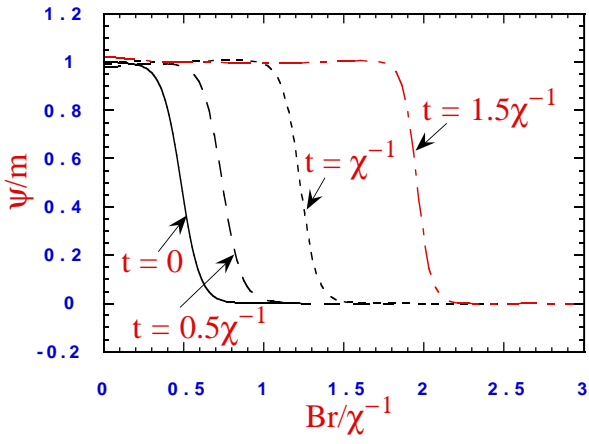


Fig.2(a)

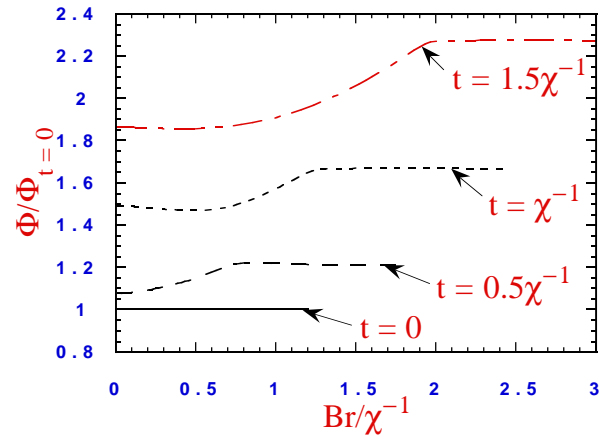


Fig.2(b)

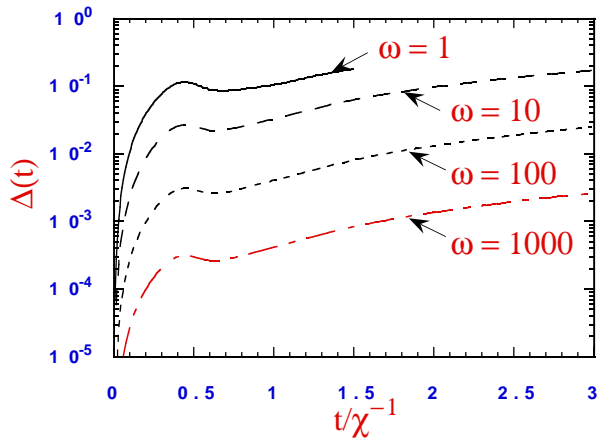


Fig.3

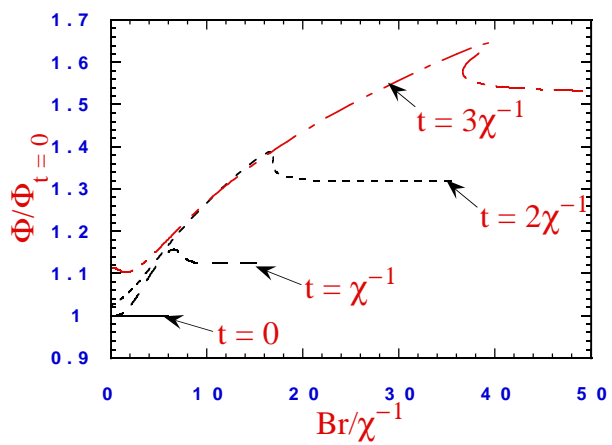


Fig.4(a)

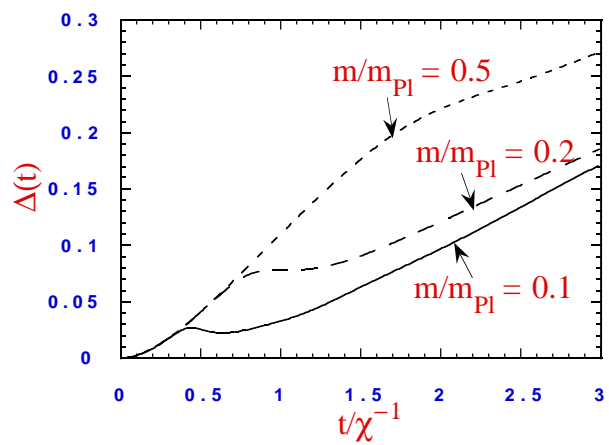


Fig.4(b)

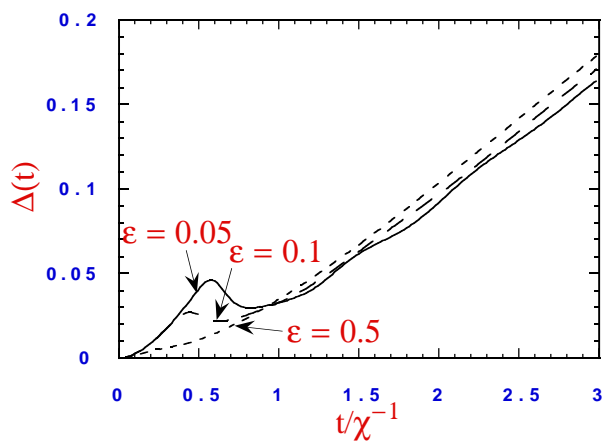


Fig.5